

A SURVEY OF NONADIABATIC SUPERCONDUCTIVITY IN CUPRATES AND FULLERIDES

EMMANUELE CAPPELLUTI

*Dipartimento di Fisica, Università "La Sapienza", P.le Aldo Moro 2
 Roma, 00185, Italy
 and INFN, Unità Roma1*

CLAUDIO GRIMALDI

*École Polytechnique Fédérale, Département de microtechnique IPM
 Lausanne, CH-1015, Switzerland*

LUCIANO PIETRONERO

*Dipartimento di Fisica, Università "La Sapienza", P.le Aldo Moro 2
 Roma, 00185, Italy
 and INFN, Unità Roma1*

and

SIGFRID STRÄSSLER

*École Polytechnique Fédérale, Département de microtechnique IPM
 Lausanne, CH-1015, Switzerland*

High- T_c superconductors are characterized by very low carrier densities. This feature leads to two fundamental consequences: on one hand the Fermi energies are correspondingly small and they can be of the same order of phonon frequencies. In such a situation nonadiabatic corrections arising from the breakdown of Migdal's theorem can not be longer neglected. In addition, small carrier densities imply poor screening and correlation effects have to be taken into account. We present a comprehensive overview of the theory of superconductivity generalized into the nonadiabatic regime which is qualitatively different from the conventional one. In this framework some of the observed properties of the cuprates and the fullerene compounds can be naturally accounted for, and a number of theoretical predictions are proposed that can be experimentally tested.

After almost 15 years and in spite of the huge amount of work spent in the field, no definitive description of the high temperature superconductivity phenomenon has been still achieved. In the meanwhile, a lot of "exotic" features have been discovered to compose the extremely rich phase diagram of these compounds. Most recently, theoretical and experimental research has addressed the issues of the pseudogap onset and of a possible stripe ordering. Many models have been proposed in order to account at least for some of the several anomalies in the high temperature superconductivity compounds (HTSC). Among them, the role of the electron-phonon (*el-ph*) coupling has received alternate fortune. In this contribution we review the

Table 1. Characteristic features of conventional superconductors compared with the high- T_c materials.

conventional materials	HTSC
$T_c^{\max} \sim 20$ K	$T_c \sim 40 \div 100$ K
$\alpha_{T_c} = 0.5$	$\alpha_{T_c} \sim 0.1 \div 0.8$
$\alpha_{m^*} = 0$	$\alpha_{m^*} \sim -0.6 \div -0.8$
$\rho(T) \propto T^5$	$\rho(T) \propto T$
<i>s</i> wave	<i>d</i> wave
large bands (high density of charge carriers)	narrow bands (low density of charge carriers)
phononic pairing	pairing (?)

main experimental evidences of a relevant role of the *el-ph* interaction on the superconductive pairing. We discuss these evidences in the context of the nonadiabatic theory of superconductivity and of the normal state.^{1,2,3} The anomalies of the *el-ph* phenomenology, which were interpreted as hints of negligible conventional *el-ph* interaction, acquire now a natural explanation in the nonadiabatic regime. We also point briefly out analogies and differences between the cuprate family and the fullerides.

From the point of view of the conventional theory of superconductivity, described by Migdal-Eliashberg (ME) equations, the high value of the critical temperature T_c in the HTSC is by itself a puzzle. The well-known McMillan formula relates T_c essentially to two microscopic parameters, Ω_b and λ_b , which represent respectively the energy scale of the intermediate boson and its coupling strength with electrons. In order to achieve in conventional theory T_c 's as high as 100 K one should assume an anomalous large λ_b , physically prevented by structural distortions, or alternatively high energy bosons such as electronic excitations. This latter idea was initially supported by the discovery of a negligible isotope effect on T_c , α_{T_c} , at optimal doping. However, later works found a drastic increase of α_{T_c} , up to $\alpha_{T_c} \simeq 0.8$, as the materials were underdoped.⁴ Moreover, recent studies reported a finite, large and negative isotope effect on the electronic mass $\alpha_{m^*} \sim -0.6 \div -0.8$, whereas $\alpha_{m^*} = 0$ is expected in conventional ME theory.⁵ These experimental results point out the relevance of *el-ph* scattering in determining superconducting and normal state properties. The small value of α_{T_c} as well as the linear dependence

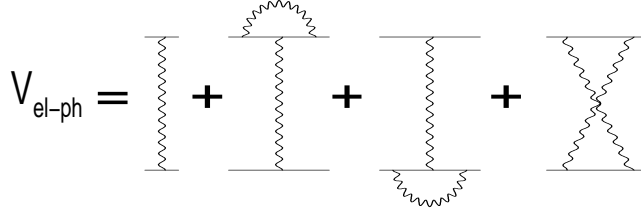


Fig. 1. Particle-particle interaction in nonadiabatic regime including corrections beyond Migdal's theorem.

on temperature of the resistivity, both peculiarities of the optimal doping, could be understood better as signature of some anomalous feature of optimal doping on the top of a phononic pairing scenario rather than characteristic of the whole phase diagram. Is d -wave symmetry of the superconductive gap compatible with this outlined picture? Conventional $el-ph$ pairing does not usually show any momentum structure, yielding an isotropic s -wave superconductivity. However, it has been shown by different techniques that strong electronic correlation induces a momentum structure with a predominance of forward scattering (small \mathbf{q} 's).⁶ In this situation d -wave symmetry can be favoured with respect to s -wave even within a phonon driven superconductivity.

As it is clear from the above brief overview of experimental results, $el-ph$ coupling seems to play a major role in the normal and superconductive phenomenology, but at the same time it can not be properly understood within the framework of conventional ME theory. The new perspective we propose is the nonadiabatic theory of Fermi liquid.^{1,2,3}

Electronic structure in high- T_c materials is characterized by narrow bands crossing the Fermi level. This situation has two fundamental consequences with respect to both phononic and electronic scattering: (i) on one hand, electronic bands in these compounds can be so narrow to be of the same order of the phonon energy scale; (ii) on the other hand, strong electronic correlation effects are also definitively important in narrow band systems. Theoretical works has focused mainly on the second point. In our opinion, the first one can be equally and even more important.

The adiabatic parameter, defined as ratio between phonon frequencies and Fermi energy Ω_{ph}/E_F , can be as large as $\Omega_{ph}/E_F \sim 0.3$ in cuprates. In this regime Migdal's theorem, on which conventional $el-ph$ ME theory rests, breaks down.⁷ In order to investigate the nonadiabatic regime, in the past years we have developed a new theory using a different perturbation approach based on $\lambda\Omega_{ph}/E_F$ instead of Ω_{ph}/E_F .^{1,2,3} A sketch of the Cooper channel interaction in the nonadiabatic regime is depicted in Fig. 1. A point which we would like to stress is that nonadiabaticity defines a *qualitatively* new theory wherein $el-ph$ interaction plays a different role than in conventional ME theory. In this context, strong correlation acts as a positive factor with respect to $el-ph$ coupling. As we have shown, the forward scattering predominance induced by the strong electronic correlation selects the phase space

where nonadiabatic effects enhance the effective *el-ph* interaction.

In a schematized picture, we can say nonadiabaticity provides a enlarged scenario where corrections to the conventional ME theory, arising from the breakdown of Migdal's theorem, can favour or depress superconducting pairing depending on the degree of electronic correlation in the system. This latter can be parametrized by the quantity q_c representing the momentum selection induced by the strong correlation.¹ More correlated the system smaller q_c .⁶

The issue of a phonon or non-phonon driven superconductivity is now re-analyzed in the light of the nonadiabatic theory of superconductivity. The evidences previously discussed of an “anomalous” *el-ph* pairing can now be understood in a natural way in the context of phonon mediated nonadiabatic superconductivity sustained by strong electronic correlation.

- T_c : critical temperatures as large as the experimental ones (up to 100 K) are accompanied in HTSC by a strong coupling phenomenology (large ratio $2\Delta/T_c$, anomalous dip in tunnelling, etc ...). Conventional phonon based ME theory requires an unphysically large *el-ph* coupling ($\lambda \sim 4$) to account for these features.⁸ In several works we have shown as nonadiabatic corrections modify the *el-ph* interaction and the structure itself of the Eliashberg equations generalized in nonadiabatic regime. For small q_c 's (strong correlation) nonadiabatic corrections enhance the effective *el-ph* pairing reproducing high critical temperatures and strong coupling phenomenology with reasonable values of λ ($\lambda \sim 1$).^{9,10}
- α_{T_c} : By simple scaling analyses, the isotope effect α_{T_c} in ME theory (with no Coulomb repulsion: $\mu = 0$) is shown to be $\alpha_{T_c} = 0.5$ for *any* λ . However, as above discussed, nonadiabatic theory does not yield just an “effective” enhanced *el-ph* coupling λ , but defines a qualitatively new theory where a strong coupling phenomenology arises from normal value of λ . In similar way, evaluation of the isotope effect is also deeply different in nonadiabatic superconductivity. In particular, α_{T_c} shows strong fluctuations ($\alpha_{T_c} \sim 0.2 - 0.8$ for $\mu = 0$) for small q_c 's, precisely where the enhancement of T_c is the largest, in agreement with measurement data.^{2,3} An additional role can be played by the Van Hove singularity experimentally observed.¹¹
- α_{m^*} : the different structure of the nonadiabatic equations of superconductivity is reflected also in a finite and negative isotope effect on the effective electronic mass m^* ,¹² as experimentally observed.⁵ This can be therefore considered as a trademark of nonadiabaticity. Other possible explanations, as polaron band narrowing or closeness of a Van Hove singularity, do not seem satisfactory, although Van Hove singularity is certainly present.
- $\rho(T) \propto T$: one of the most striking features of HTSC is the linear dependence of $\rho(T)$ in a wide range of temperature (up to 1100 K in La214).¹³ At so high temperature resistivity is expected to be dominated by phonon scattering. A linear resistivity with no change of slope suggests therefore a common, phonon based, scattering mechanism for high and low temperature. As we have shown

in Ref. 11, the linear behaviour can be related to the presence of a Van Hove singularity located at optimal doping in the context of nonadiabatic phonon scattering.¹¹

- *d*-wave: it is often argued in literature that *d*-wave superconductivity is not compatible with phonon pairing while it points out a spin fluctuation interaction. Reason for this belief is the structureless *el-ph* interaction of conventional ME materials which leads to *s*-wave symmetry. HTSC compounds, however, are characterized by strong correlation inducing an important momentum selection with predominance of forward scattering. In such a situation *d*-wave superconductivity is favoured with respect to *s*-wave. We find a crossover from *s*- to *d*-wave symmetry by lowering the momentum selection q_c , or, in other words, by increasing the rate of electronic correlation. Nonadiabatic effects are shown to increase both the kind of ordering.¹⁴
- low charge carrier density: Another puzzling feature of HTSC materials, as shown by Uemura's plot,¹⁵ is the extreme low density of charge carriers, at least one order of magnitude lower than in low temperature superconductors. It is therefore quite surprising that the best superconductors are those with the poorest number of carriers whereas in conventional ME theory T_c increases with the number of carriers through the density of states. This inconsistency is solved in natural way in the nonadiabatic theory of superconductivity. Small number of carriers leads to small Fermi energies driving the system into nonadiabatic regime where correction beyond Migdal's theorem become important. Moreover, low density of charges implies poor screening of long-range *el-ph* interactions (small q_c 's) and predominance of small exchanged momenta.

From the above analysis of different “exotic” features of HTSC materials the fundamental role of *el-ph* interaction appears evident. However, these feature can not be properly explained in the conventional framework of ME theory. The nonadiabatic theory of superconductivity and normal state provides a coherent picture where the above discussed anomalies arise as natural hallmarks of nonadiabatic effects. The primary actors are two: on one hand small Fermi energies determine the nonadiabaticity rate of the system and turn on nonadiabatic corrections due to the breakdown of Migdal's theorem. Within this enriched phonon based scenario an additional but fundamental role is played by the strong electronic correlation that, through the induced momentum selection, amplifies the nonadiabatic effects and yields an effective enhanced *el-ph* coupling. This modified *el-ph* interaction, generalized in nonadiabatic regime, defines a new theory qualitatively different both from the conventional ME one and from the polaronic picture. Secondary effects can be also related to supporting actors, as the Van Hove singularity, that need in any case to be considered to account for peculiar details on the phase diagram.

All through this contribution, we have mainly focused on the physics of cuprates, the most studied HTSC compounds by theoretical and experimental means. However, a look at Uemura's plot, completed by later materials like the fullerenes, suggests a common origin of superconductivity for the different families of “HTSC”

materials, as cuprates, bismuthates, fullerides and heavy fermion systems.¹⁵ All these compounds are characterized by small density of charge carriers in contrast to conventional ME materials, and nonadiabaticity stands out as the natural candidate to explain superconductivity in these systems. The relevant parameter will be the ratio between intermediate boson frequencies (spin fluctuations in heavy fermions or phonons in fullerides and bismuthates) and the Fermi energies. In particular, alkali-doped C₆₀ compounds appear, for their relatively simple phase diagram, as the best materials where to check the nonadiabatic Fermi liquid picture. A detailed study shows how the conventional ME theory can not explain the experimental data available for these materials (high values of T_c , of $2\Delta/T_c$, small value of αT_c) while the nonadiabatic theory is able to reproduce the experimental scenario with quite realistic values of the microscopical parameters.¹⁰ In addition, our proposed description is liable to be tested in experimental different ways. For instance, we predict a finite isotope effect on the electronic specific heat¹² and on the spin susceptibility¹⁶ in fullerides and more generally in any nonadiabatic superconductor. An anomalous reduction of T_c by paramagnetic impurity scattering is also expected.¹⁷ Experimental accuracy for these kind of measurements in nowadays already available and any experimental research along this line is welcome.

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